Nodal Analysis (DC Excitation)

Note: Procedure to solve problems on nodal Analysis (Both DC & AC excitations)

i) If nodes are not specified in the problem, identify Principal nodes (Where three or more branches meet). One node should be reference node which is of zero potential. Name the node voltages as $V_1, V_2, V_3 \ldots$

ii) To identify essential node, non-essential node & supernode in place of Voltage Source (—(+)—) put a short circuit. A Voltage source in series with a resistance (—(+)—ww—) should not be put as a short circuit.

iii) • → Essential node, not merging with any other node. We write KCL equation.

—⊥ → Non-essential node, merging with reference node. We write KVL equation.

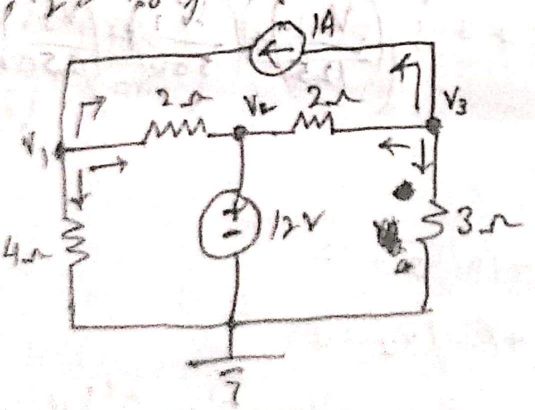
•—• → Super node, nodes are merging but none of it is reference node. We write KCL & KVL equation

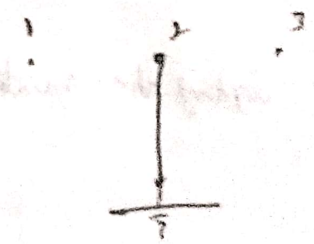
iv) Equations are sorted out & by making use of Calculator, solve the simultaneous equations for the unknowns $V_1, V_2, V_3 \ldots$

v) Current through the elements and power can be obtained by node voltages.

Prob: Find V_1, V_2 & V_3 for the circuit shown in figure using node analysis.

Sol:





KVL equation for non-essential node 2: $V_2 - 0 = 12 \Rightarrow V_2 = 12$

KCL equation for essential node 1: $\left(\frac{V_1}{4}\right) + \left(\frac{V_1 - V_2}{2}\right) - 1 = 0$

KCL equation for essential node 3: $\left(\frac{V_3}{3}\right) + \left(\frac{V_3 - V_2}{2}\right) + 1 = 0$

Sorting the equations

$$(0)V_1 + (1)V_2 + (0)V_3 = 12$$

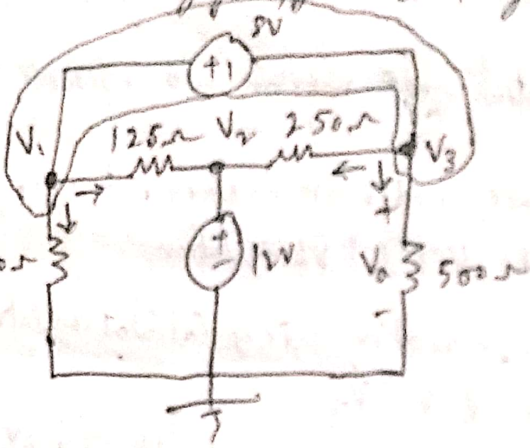
$$(4^{-1}+2^{-1})V_1 + (-2^{-1})V_2 + (0)V_3 = 1$$

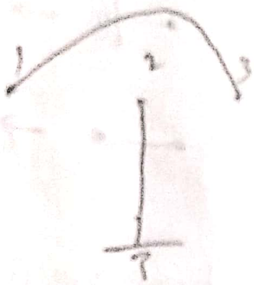
$$(0)V_1 + (-2^{-1})V_2 + (3^{-1}+2^{-1})V_3 = -1$$

$$V_1 = 9.33\,V$$
$$V_2 = 12\,V$$
$$V_3 = 6\,V$$

Prob: For the circuit shown in figure find V_a using nodal analysis.

Sol:





KVL equation for non-essential node, $V_2 = 12$

KVL equation for super node, $V_1 - V_3 = 8$

KCL equation for supernode 1 & 3 : $\left(\frac{V_1 - V_2}{125}\right) + \left(\frac{V_1}{500}\right) + \left(\frac{V_3 - V_2}{250}\right) + \left(\frac{V_3}{500}\right) =$

Sorting the equations

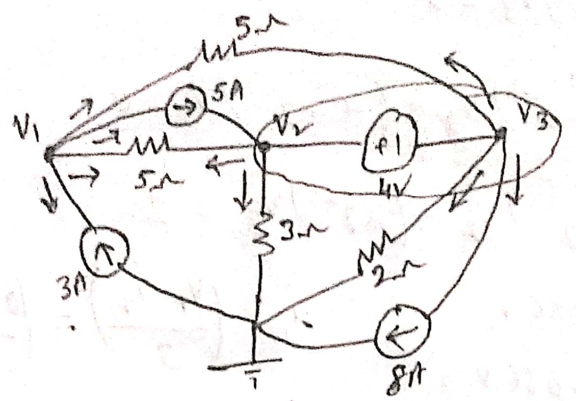
$$(0)V_1 + (1)V_2 + (0)V_3 = 12$$

$$(1)V_1 + (0)V_2 + (-1)V_3 = 8$$

$$(125^{-1} + 500^{-1})V_1 + (-125^{-1} - 250^{-1})V_2 + (250^{-1} + 500^{-1})V_3 = 0$$

$$V_1 = 12\,V$$
$$V_2 = 12\,V$$
$$V_a = V_3 = 4\,V$$

prob. For the circuit shown in figure determine all the node voltages.



KVL to super node 2 & 3 : $V_2 - V_3 = 4$

KCL to super node 2 & 3 : $\left(\frac{V_3 - V_1}{5}\right) + 8 + \left(\frac{V_3}{2}\right) + \left(\frac{V_2}{3}\right) + \left(\frac{V_2 - V_1}{5}\right) - 5 = 0$

KCL to essential node 1 : $-3 + \left(\frac{V_1 - V_2}{5}\right) + 5 + \left(\frac{V_1 - V_3}{5}\right) = 0$

$$(0)V_1 + (1)V_2 + (-1)V_3 = 4$$

$$(-5^{-1} - 5^{-1})V_1 + (3^{-1} + 5^{-1})V_2 + (5^{-1} + 2^{-1})V_3 = -3$$

$$(5^{-1} + 5^{-1})V_1 + (-5^{-1})V_2 + (-5^{-1})V_3 = -2$$

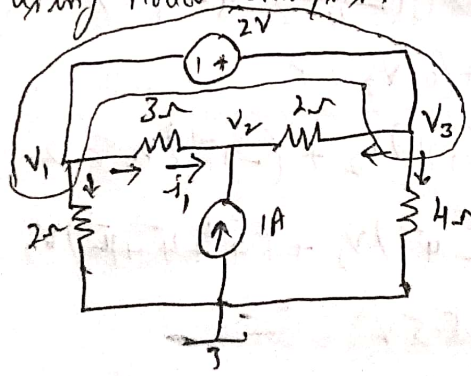
$V_1 = -10.6\,V$

$V_2 = -3.6\,V$

$V_3 = -7.6\,V$

Prob: Find i_1 using nodal analysis.

Sol:



KVL to super node 1 & 3 : $V_3 - V_1 = 2$

KCL to super node 1 & 3 : $\left(\frac{V_1}{2}\right) + \left(\frac{V_1 - V_2}{3}\right) + \left(\frac{V_3}{4}\right) + \left(\frac{V_3 - V_2}{2}\right) = 0$

KCL to essential node 2 : $\left(\frac{V_2 - V_1}{3}\right) - 1 + \left(\frac{V_2 - V_3}{2}\right) = 0$

$(-1)V_1 + (0)V_2 + (1)V_3 = 2$

$(2^{-1}+3^{-1})V_1 + (-3^{-1}-2^{-1})V_2 + (4^{-1}+2^{-1})V_3 = 0$

$(-3^{-1})V_1 + (3^{-1}+2^{-1})V_2 + (-2^{-1})V_3 = 1$

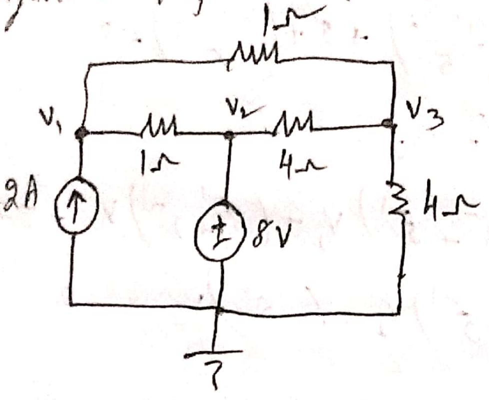
$V_1 = 0.666\ V$ $\quad i_1 = \left(\frac{V_1 - V_2}{3}\right) = \left(\frac{0.666 - 3.066}{3}\right)$

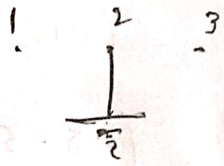
$V_2 = 3.066\ V$ $\quad i_1 = -0.8A$

$V_3 = 2.666V$

Prob: Apply node analysis method to find node voltages V_1, V_2, V_3 for the circuit shown in the figure. Find the power dissipated in all the resistances.

Sol:





KVL to non-essential node 2: $V_2 = 8$

KCL to essential node 1: $\left(\frac{V_1 - V_2}{1}\right) + \left(\frac{V_1 - V_3}{1}\right) - 2 = 0$

KCL to essential node 3: $\left(\frac{V_3}{4}\right) + \left(\frac{V_3 - V_2}{4}\right) + \left(\frac{V_3 - V_1}{1}\right) = 0$

$(0)V_1 + (1)V_2 + (0)V_3 = 8$

$(1^{-1}+1^{-1})V_1 + (-1^{-1})V_2 + (-1^{-1})V_3 = 2$

$(-1^{-1})V_1 + (-4^{-1})V_2 + (4^{-1}+4^{-1}+1^{-1})V_3 = 0$

$V_1 = 8.5V$

$V_2 = 8V$

$V_3 = 7V$

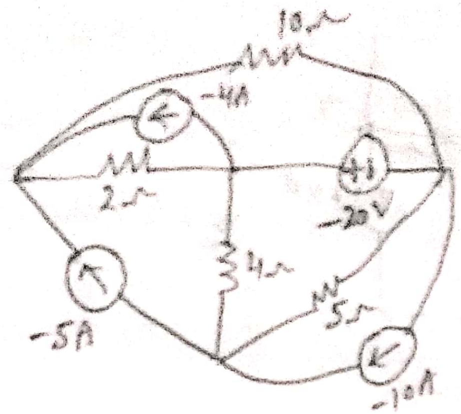
$P = \frac{V^2}{R}$

$P_{1\Omega} = \frac{(V_1 - V_2)^2}{1} = 0.25W$ $\quad P_{1\Omega} = \frac{(V_1 - V_3)^2}{1} = 2.25W$

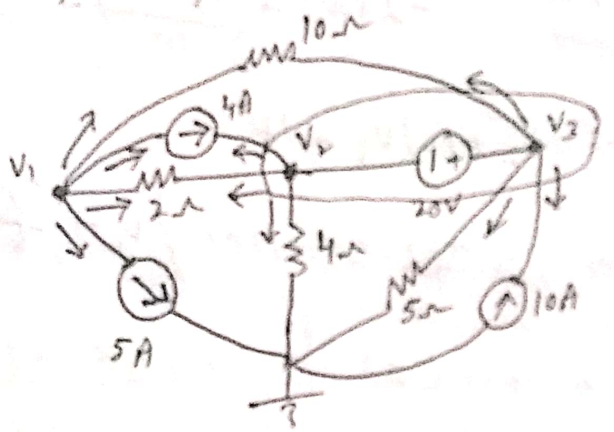
$P_{4\Omega} = \frac{(V_2 - V_3)^2}{4} = 0.25W$

$P_{4\Omega} = \frac{V_3^2}{4} = 12.25W$

Prob: Find the node voltages in the circuit shown in the figure



Sol: change the direction of current source to make the magnitude of current +ve.
change the polarity of voltage source to make the magnitude of voltage +ve



KVL to supernode 2 & 3 : $V_3 - V_2 = 20$

KCL to supernode 2 & 3 : $\left(\frac{V_2}{4}\right) + \left(\frac{V_2 - V_1}{2}\right) - 4 - 10 + \left(\frac{V_3}{5}\right) + \left(\frac{V_3 - V_1}{10}\right) = 0$

KCL to essential node 1 ; $5 + \left(\frac{V_1 - V_2}{2}\right) + 4 + \left(\frac{V_1 - V_3}{10}\right) = 0$

$$(0)V_1 + (-1)V_2 + (1)V_3 = 20$$

$$(-2^{-1} - 10^{-1})V_1 + (4^{-1} + 2^{-1})V_2 + (5^{-1} + 10^{-1})V_3 = 14$$

$$(2^{-1} + 10^{-1})V_1 + (-2^{-1})V_2 + (-10^{-1})V_3 = -9$$

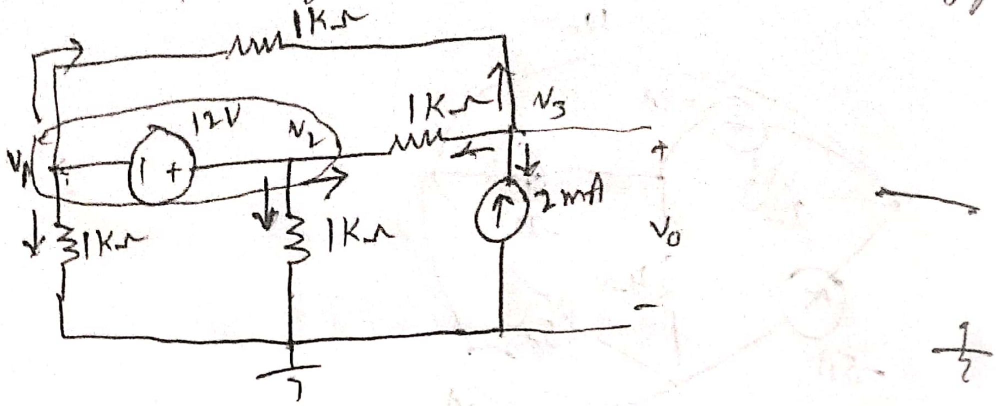
$V_1 = -9.44\text{V}$

$V_2 = 2.22\text{V}$

$V_3 = 22.22\text{V}$

Prob: Use the nodal analysis to find V_0 in the network shown in figure.

Sol:



KVL to supernode 1 & 2 : $V_2 - V_1 = 12$

KCL to supernode 1 & 2 ; $\left(\frac{V_1}{1K}\right) + \left(\frac{V_1 - V_3}{1K}\right) + \left(\frac{V_2}{1K}\right) + \left(\frac{V_2 - V_3}{1K}\right) =$

KCL to essential node 3 ; $-2\times10^{-3} + \left(\frac{V_3 - V_2}{1K}\right) + \left(\frac{V_3 - V_1}{1K}\right) = 0$

Note:
use $-(1K)^{-1}$
minus outside
brackets to take inverse

$$(-1)V_1 + (1)V_2 + (0)V_3 = 12$$

$$\left\{(1K)^{-1} + (1K)^{-1}\right\}V_1 + \left\{(1K)^{-1} + (1K)^{-1}\right\}V_2 + \left\{-(1K)^{-1} - (1K)^{-1}\right\}$$

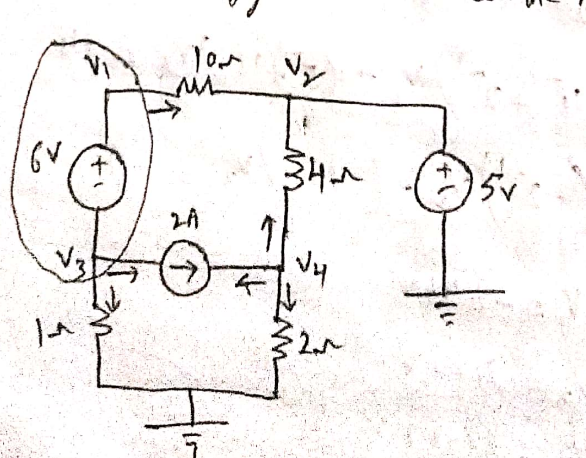
$$\left\{-(1K)^{-1}\right\}V_1 + \left\{-(1K)^{-1}\right\}V_2 + \left\{(1K)^{-1} + (1K)^{-1}\right\}V_3 = 2$$

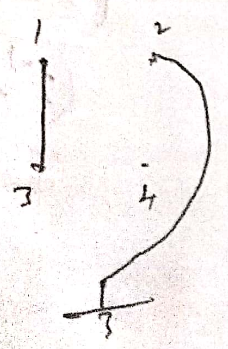
While using calculator substitute 10^3 for K

$$V_1 = -5V$$
$$V_2 = 7V$$
$$V_0 = V_3 = 2V$$

Prob: For the circuit shown in the figure determine all the node voltages.

Sol:





KVL to non-essential node : $V_2 = 5$

substitute $V_2 = 5$ in the following equations & the unknowns will be, V_1, V_3, V_4

KVL to supernode 1 & 3 : $V_1 - V_3 = 6$

KCL to supernode 1 & 3 : $\left(\frac{V_1 - \overset{\to V_2}{5}}{10}\right) + \left(\frac{V_3}{1}\right) + 2 = 0$

KCL to essential node 4 : $\left(\frac{V_4 - \overset{\to V_2}{5}}{4}\right) - 2 + \left(\frac{V_4}{2}\right) = 0$

$$(1)V_1 + (-1)V_3 + (0)V_4 = 6$$

$$(10^{-1})V_1 + (1^{-1})V_3 + (0)V_4 = 5 \times 10^{-1} - 2$$

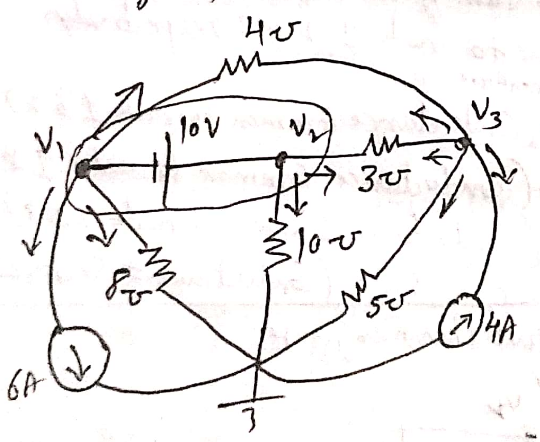
$$(0)V_1 + (0)V_3 + (4^{-1} + 2^{-1})V_4 = 5 \times 4^{-1} + 2$$

$V_1 = 4.09\ V$

$V_3 = -1.909\ V$ $\quad V_2 = 5\ V$

$V_4 = 4.333\ V$

Prob: Find the node voltages for the circuit shown in the figure.



Note: All the resistances are given in ℧ (mho) (old unit) or S (siemens) (new unit) } Conductance (G)

$I = \frac{V}{R}$

$I = VG$

KVL for supernode 1 & 2 : $V_2 - V_1 = 10$

KCL for supernode 1 & 2 : $6 + 8V_1 + 4(V_1 - V_3) + 3(V_2 - V_3) + 10V_2 = 0$

KCL for essential node 3 : $-4 + 5V_3 + 3(V_3 - V_2) + 4(V_3 - V_1) = 0$

1 2 3

$$(-1)V_1 + (1)V_2 + (0)V_3 = 10$$
$$(12)V_1 + (13)V_2 + (-7)V_3 = -6$$
$$(-4)V_1 + (-3)V_2 + (12)V_3 = 4$$

$$V_1 = -5.55\ V$$
$$V_2 = 4.44\ V$$
$$V_3 = -0.406\ V$$

Inspection method

When all the nodes are essential nodes, then inspection method is used (i.e. circuit with no ideal voltage source (—(+)—)

It is based on $[G][V] = [I]$

For a three node circuit,

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

I_1, I_2, I_3 → Current to node 1, 2, 3 respectively

Current entering node → +ve

current leaving node → −ve

V_1, V_2, V_3 → unknown node voltages.

G_{11}, G_{22}, G_{33} → sum of all the conductances connected to node 1, 2 & 3 respectively

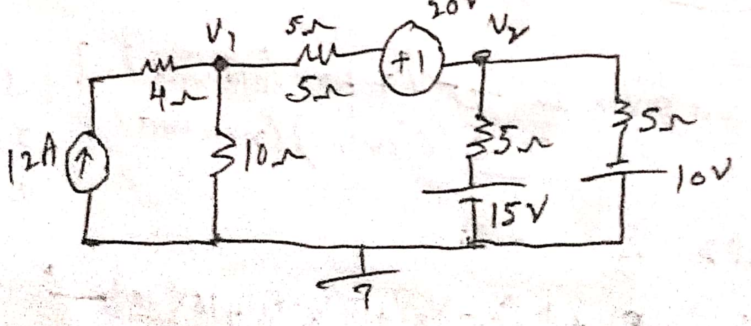
$G_{12} = G_{21}$ → −(Conductance common to node 1 & 2) (negative)

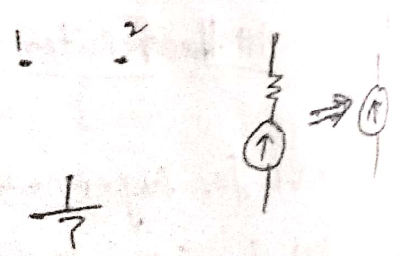
$G_{13} = G_{31}$ → −(Conductance common to node 1 & 3)

$G_{23} = G_{32}$ → −(Conductance common to node 2 & 3)

(Conductance = Reciprocal of resistance)

Prob: Find the node voltages for the circuit shown in figure.





Both the nodes are essential nodes. 4Ω in series with current source neglected

By inspection

+ towards node 1

$$(10^{-1}+5^{-1})V_1 + (-5^{-1})V_2 = +12 + \frac{20 \rightarrow V}{5 \rightarrow R}$$

$$(-5^{-1})V_1 + (5^{-1}+5^{-1}+5^{-1})V_2 = -\frac{20}{5} + \frac{15}{5} - \frac{10}{5}$$

- towards node 2

$$V_1 = 64.3\ V$$

$$V_2 = 16.43\ V$$

RHS

$\downarrow$

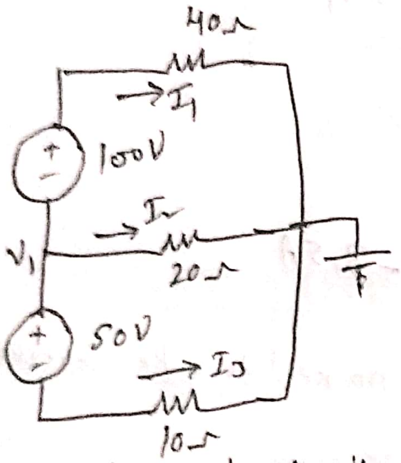
$12 + 20 \times 5^{-1}$

$-20 \times 5^{-1} + 15 \times 5^{-1}$

$- 10 \times 5^{-1}$

(While using Calculator)

Prob: Find I_1, I_2, I_3 using nodal analysis.



A single essential node circuit

$$V_1(20^{-1}+40^{-1}+10^{-1}) = \frac{-100}{40} + \frac{50}{10}$$

$$V_1 = 14.286\ V$$

opposite polarity of node 1

$$I_1 = \left(\frac{V_1+100}{40}\right) = 2.857A$$

$$I_2 = \frac{V_1}{20} = 0.714A$$

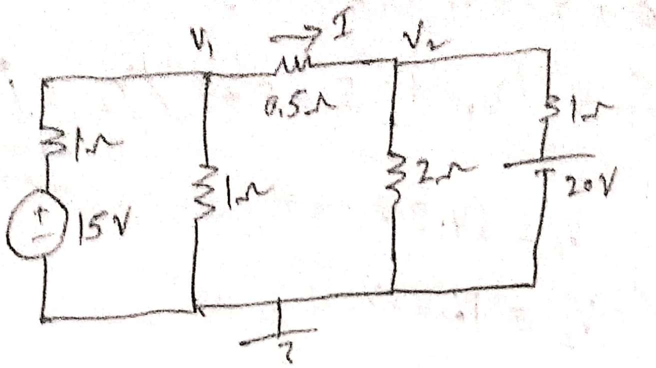
opposite polarity of node 1

$$I_3 = \left(\frac{V_1-50}{10}\right) = -3.571A$$

$$I_1 + I_2 + I_3 = 0$$

Prob: Find I using nodal analysis for the circuit shown in figure.

Sol:



By inspection method,

$$(1^{-1}+1^{-1}+0.5^{-1})V_1+(-0.5^{-1})V_2=\frac{15}{1}$$

$$(-0.5^{-1})V_1+(0.5^{-1}+2^{-1}+1^{-1})V_2=\frac{20}{1}$$

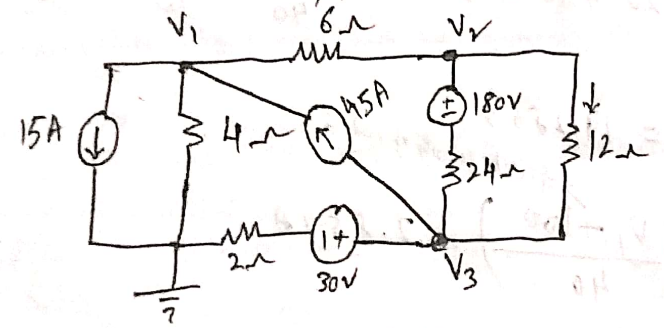
$$V_1 = 9.25\ V$$

$$V_2 = 11V$$

$$I = \frac{V_1 - V_2}{0.5} = -3.5A$$

Note: If resistances are given in ℧ or S, no need to take inverse. The given values are taken as it is.

Prob: Using nodal analysis, find the current through 12Ω



By inspection, $(4^{-1}+6^{-1})V_1+(-6^{-1})V_2+(0)V_3 = 45-15$

$$(-6^{-1})V_1+(6^{-1}+24^{-1}+12^{-1})V_2+(-24^{-1}-12^{-1})V_3=\frac{180}{24}$$

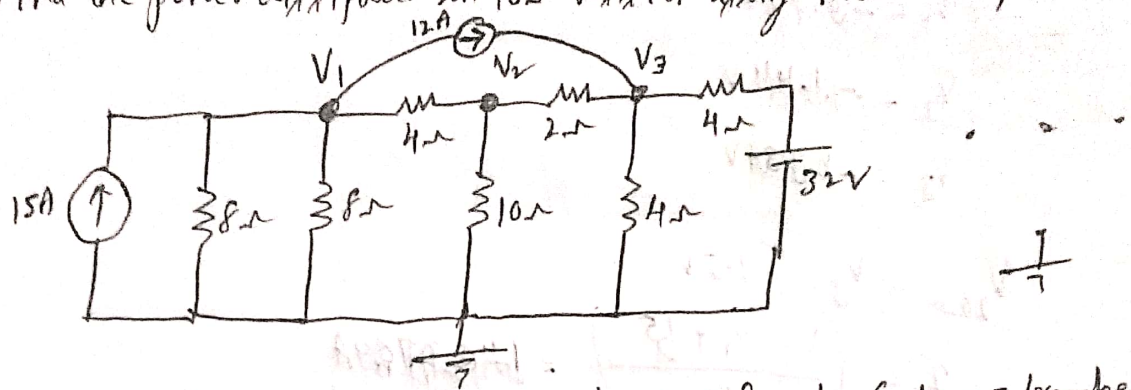
$$(0)V_1+(-24^{-1}-12^{-1})V_2+(2^{-1}+24^{-1}+12^{-1})V_3=\frac{30}{2}-45-\frac{180}{24}$$

$$V_1 = 96\ V \quad V_2 = 60\ V \quad V_3 = -48V$$

$$I_{12\Omega} = \frac{(V_2-V_3)}{12} = \left(\frac{60+48}{12}\right) = 9A$$

Prob: Find the power dissipated in 10Ω resistor using nodal analysis.

Sol:



Nodes are not given in the problem. only principal nodes (where 3 branches or more meet) are considered.

All are essential nodes. By inspection,

$$(8^{-1}+8^{-1}+4^{-1})V_1+(-4^{-1})V_2+(0)V_3=+15-12$$

$$(-4^{-1})V_1+(4^{-1}+2^{-1}+10^{-1})V_2+(-2^{-1})V_3=0$$

$$(0)V_1+(-2^{-1})V_2+(2^{-1}+4^{-1}+4^{-1})=+\frac{32}{4}+12$$

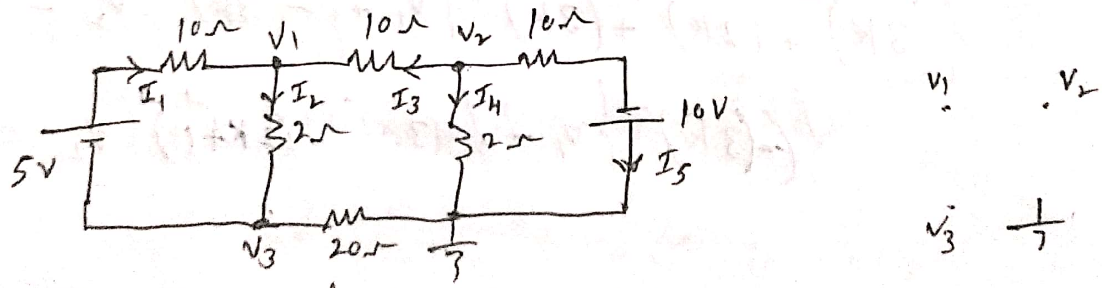
$V_1 = 18.10\,V$

$V_2 = 24.21\,V$

$V_3 = 32.10\,V$

$$P_{10\Omega}=\frac{V_2^2}{R}=V_2^2G=\frac{24.21^2}{10}=58.61\,W$$

Prob: Find Voltage across 20Ω & currents in various branches



Sol: Considering only principal nodes, by inspection,

$$(10^{-1}+10^{-1}+2^{-1})V_1+(-10^{-1})V_2+(-2^{-1}-10^{-1})V_3=\frac{5}{10}$$

$$(-10^{-1})V_1+(10^{-1}+10^{-1}+2^{-1})V_2+(-0)V_3=-\frac{10}{10}$$

$$(-2^{-1}-10^{-1})V_1+(-0)V_2+(20^{-1}+2^{-1}+10^{-1})V_3=-\frac{5}{10}$$

$V_1 = -0.741 V$

$V_2 = -1.541 V$

$V_3 = -1.5 V$

$V_{20\Omega} = V_3 = -1.5 V$

$I_1 = \left(\frac{V_3 - V_1 + 5}{10}\right) = 0.4291 A$

$I_2 = \left(\frac{V_1 - V_3}{2}\right) = 0.3545 A$

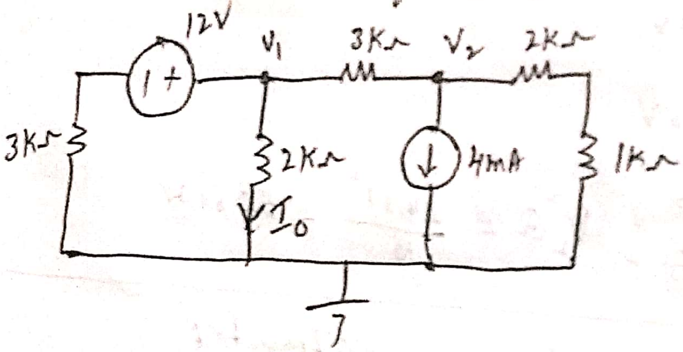
$I_3 = \left(\frac{V_2 - V_1}{10}\right) = -0.075 A$

$I_4 = \frac{V_2}{2} = -0.7705$

$I_5 = \left(\frac{V_2 + 10}{10}\right) = 0.8459 A$

Prob: Find the Current I_0 using nodal analysis.

Sol:



$$\left\{(3K)^{-1} + (2K)^{-1} + (3K)^{-1}\right\} V_1 + \left\{-(3K)^{-1}\right\} V_2 = \frac{12}{(3K)}$$

$$\left\{-(3K)^{-1}\right\} V_1 + \left\{(3K)^{-1} + (2K + 1K)^{-1}\right\} V_2 = -4\times10^{-3}$$

$V_1 = 2 V$

$V_2 = -5 V$

$I_0 = \frac{V_1}{(2K)} = \frac{2}{(2\times10^3)} = 1 mA$

Using Cramer's rule, $V_2 = \dfrac{\begin{vmatrix} (0.333-j0.133) & -5\angle 0 \\ -j0.2 & 10 \end{vmatrix}}{\begin{vmatrix} (0.333-j0.133) & -j0.2 \\ -j0.2 & (0.166-j0.133) \end{vmatrix}}$

$$V_2 = \frac{10\times(0.333-j0.133) - j0.2\times 5\angle 90}{(0.333-j0.133)\times(0.166-j0.133) - j0.2\times j0.2} = \frac{(4.33-j1.33)}{(0.077-j0.0..)}$$

$$V_2 = 40.95 + j17.82 = 44.66\angle 23.52$$

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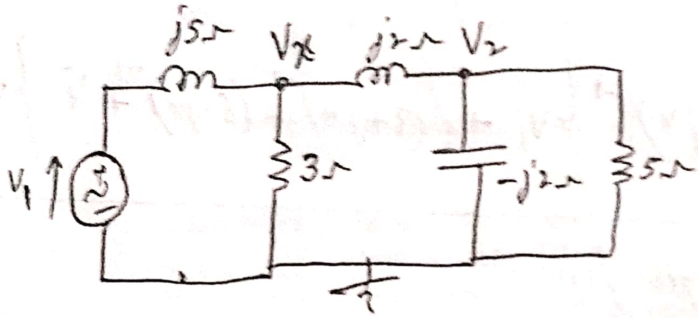
prob: Find V_1 in the circuit shown in figure using nodal analysis, when $V_2 = 20V$

Sol:



x $\frac{1}{3}$

By inspection method

$$\left\{(j5)^{-1} + 3^{-1} + (j2)^{-1}\right\}V_x + \left\{-(j2)^{-1}\right\}V_2 = \frac{V_1}{j5}$$

$$\left\{-(j2)^{-1}\right\}V_x + \left\{(j2)^{-1} + (-j2)^{-1} + 5^{-1}\right\}V_2 = 0$$

$$(0.333-j0.7)V_x + (j0.5)V_2 = -j0.2V_1$$

$$(j0.5)V_x + (0.2)V_2 = 0$$

By Cramer's rule, $V_2 = \dfrac{\begin{vmatrix} (0.333-j0.7) & -j0.2V_1 \\ j0.5 & 0 \end{vmatrix}}{\begin{vmatrix} (0.333-j0.7) & j0.5 \\ j0.5 & 0.2 \end{vmatrix}}$

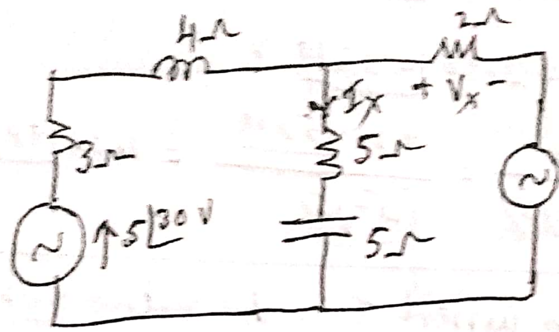
$$20 = \frac{j0.5 \times j0.2 V_1}{(0.333 - j0.7) \times 0.2 - j0.5 \times j0.5} = \frac{-0.1 V_1}{(0.3166 - j0.14)}$$

(Given)

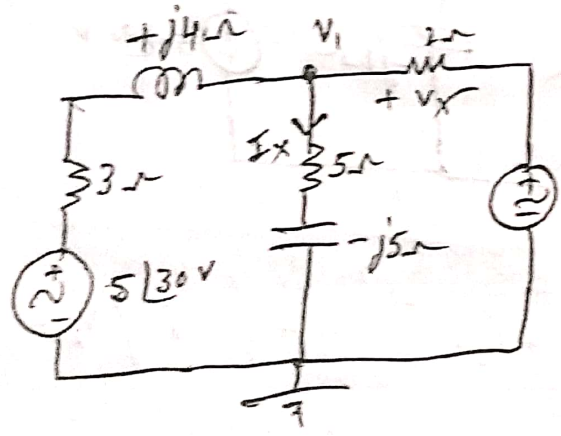
$$20 = \frac{-0.1 V_1}{(0.3166 - j0.14)}$$

$$V_1 = \frac{20 \times (0.3166 - j0.14)}{-0.1} = -63.32 + j28 = 69.23\angle 156.14$$

Prob: Use node voltage analysis, find V_x and I_x of the circuit shown in figure.



Sol: Attach +j for inductive reactance and −j for capacitive reactance which is not given in the problem. Tip: if the arrowmark is +ve for voltage source



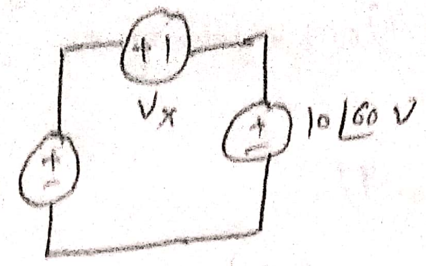
BY inspection method,

$$\left\{(3+j4)^{-1} + (5-j5)^{-1} + 2^{-1}\right\} V_1 = \frac{5\angle 30}{(3+j4)} + \frac{10\angle 60}{2}$$

$$(0.72 - j0.06) V_1 = (3.419 + j3.937)$$

$$V_1 = \frac{(3.419 + j3.937)}{(0.72 - j0.06)} = 7.217\angle 53.19 \text{ V}$$

To find V_x



KVL: $-V_1 + V_x + 10\angle 60 = 0$

$$V_x = V_1 - 10\angle 60$$

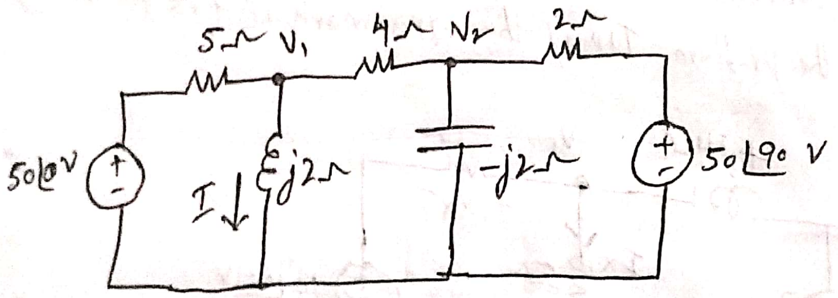
$$V_x = 7.217\angle 53.19 - 10\angle 60$$

$$V_x = 2.96\angle -103.19 \text{ V}$$

$$I_x = \frac{V_1}{(5-j5)} = \frac{7.217\angle 53.19}{(5-j5)} = 1.02\angle 98.19 \text{ A}$$

Prob: Use nodal analysis to obtain current I in the network shown in the figure.

Sol:



By inspection,

$$\left\{5^{-1} + 4^{-1} + (j2)^{-1}\right\} V_1 + \left\{-4^{-1}\right\} V_2 = \frac{50\angle 0}{5}$$

$$\left\{-4^{-1}\right\} V_1 + \left\{4^{-1} + 2^{-1} + (-j2)^{-1}\right\} = \frac{50\angle 90}{2}$$

$$(0.45 - j0.5) V_1 + (-0.25) V_2 = 10$$

$$(0.25) V_1 + (0.75 + j0.5) V_2 = 25\angle 90$$

By Cramer's rule,

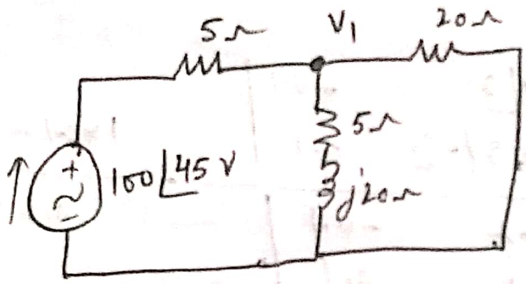
$$V_1 = \frac{\begin{vmatrix} 10 & -0.25 \\ 25\angle 90 & (0.75+j0.5) \end{vmatrix}}{\begin{vmatrix} (0.45-j0.5) & -0.25 \\ -0.25 & (0.75+j0.5) \end{vmatrix}}$$

$$V_1 = \frac{10\times(0.75+j0.5) + 25\angle 90 \times 0.25}{(0.45-j0.5)\times(0.75+j0.5) - 0.25^2}$$

$$V_1 = \frac{(7.5+j11.25)}{(0.525-j0.15)} = 24.76\angle 72.25\ V$$

$$I = \frac{V_1}{j2} = \frac{24.76\angle 72.25}{j2} = 12.38\angle -17.75\ A$$

Prob: For the network shown in figure find V_{AB} using nodal method.



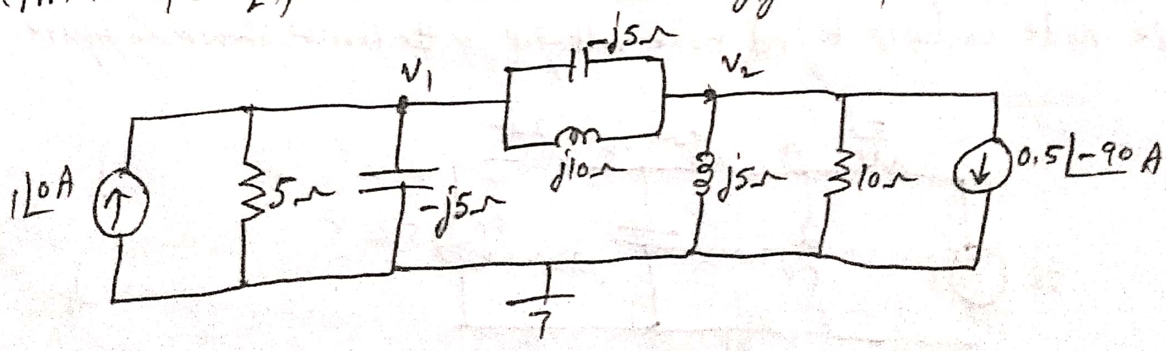
Sol: By inspection, $\left\{5^{-1} + 20^{-1} + (5+j20)^{-1}\right\} V_1 = \frac{100\angle 45}{5}$

$$(0.2617 - j0.047)V_1 = 20\angle 45$$

$$V_{AB} = V_1 = \frac{20\angle 45}{(0.2617-j0.047)} = 75.21\angle 55.18$$

Prob: Determine V_1 & V_2 for the circuit shown in figure by nodal analysis

Sol:



By inspection method,

$$\{5^{-1}+(-j5)^{-1}+(-j5)^{-1}+(j10)^{-1}\}V_1+\{-((j10)^{-1}+(-j5)^{-1})\}V_2=1\angle 0$$

$$\{-((j10)^{-1}+(-j5)^{-1})\}V_1+\{10^{-1}+(j5)^{-1}+(j10)^{-1}+(-j5)^{-1}\}V_2=-0.5\angle 90$$

$$(0.2)V_1+(-0.1)V_2=1\angle 0$$

$$(-0.1)V_1+(0.1)V_2=-0.5\angle -90$$

By Cramer's rule,

$$V_1=\frac{\begin{vmatrix}1\angle 0 & -0.1\\ -0.5\angle -90 & 0.1\end{vmatrix}}{\begin{vmatrix}0.2 & -0.1\\ -0.1 & 0.1\end{vmatrix}}=\frac{1\times 0.1-0.5\angle -90\times 0.1}{0.2\times 0.1-0.1\times 0.1}$$

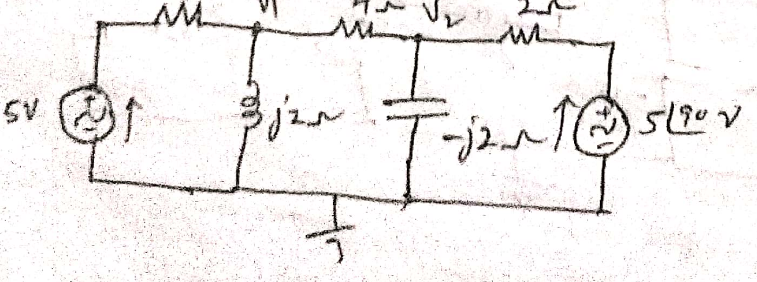
$$V_1=\frac{(0.1+j0.05)}{0.01}=11.18\angle 26.56\ V$$

$$V_2=\frac{\begin{vmatrix}0.2 & 1\angle 0\\ -0.1 & -0.5\angle -90\end{vmatrix}}{0.01}=\frac{-0.2\times 0.5\angle -90+0.1}{0.01}$$

$$V_2=\frac{(0.1+j0.1)}{0.01}=14.14\angle 45\ V$$

Pr 4: Use node analysis to find node voltages for the circuit shown in figure.

Sol:



By inspection,

$$\{5^{-1}+4^{-1}+(j2)^{-1}\}V_1 + \{-(4)^{-1}\}V_2 = \frac{5}{5}$$

$$\{-(4)^{-1}\}V_1 + \{4^{-1}+2^{-1}+(-j2)^{-1}\}V_2 = \frac{5\angle 90}{2}$$

$$(0.45-j0.5)V_1 + (-0.25)V_2 = 1$$

$$(-0.25)V_1 + (0.75+j0.5)V_2 = 2.5\angle 90$$

By cramer's rule,

$$V_1 = \frac{\begin{vmatrix} 1 & -0.25 \\ 2.5\angle 90 & (0.75+j0.5) \end{vmatrix}}{\begin{vmatrix} (0.45-j0.5) & -0.25 \\ -0.25 & (0.75+j0.5) \end{vmatrix}} = \frac{(0.75+j1.125)}{(0.525-j0.15)}$$

$$V_1 = 2.47\angle 72.25 \text{ V}$$

$$V_2 = \frac{\begin{vmatrix} (0.45-j0.5) & 1 \\ -0.25 & 2.5\angle 90 \end{vmatrix}}{(0.525-j0.15)} = \frac{(1.5+j1.125)}{(0.525-j0.15)}$$

$$V_2 = 3.43\angle 52.81 \text{ V}$$

Prob: The node voltage equations for an electric circuit as follows:

$$\left(\frac{1}{5}+\frac{1}{2}j+\frac{1}{4}\right)V_1 - \frac{1}{4}V_2 = \frac{50\angle 0}{5}$$

$$-\frac{1}{4}V_1 + \left(\frac{1}{4}-\frac{1}{j2}+\frac{1}{2}\right)V_2 = \frac{50\angle 90}{2}$$

Draw the circuit.

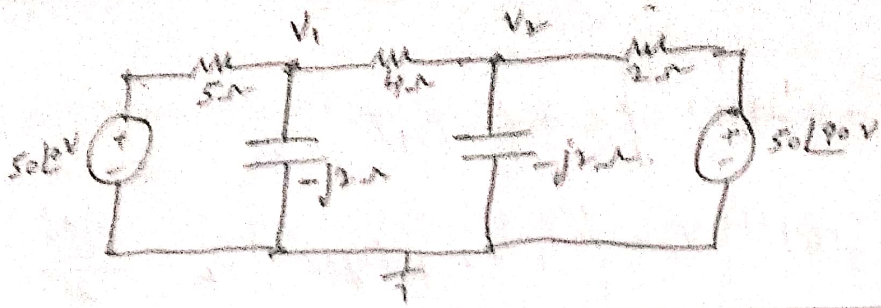
Sol:

$$G_{11} = \left(\frac{1}{5}+\frac{j}{2}+\frac{1}{4}\right) = \left(\frac{1}{5}+\frac{1}{-j2}+\frac{1}{4}\right)$$

$$G_{22} = \left(\frac{1}{4}-\frac{1}{j2}+\frac{1}{2}\right) = \left(\frac{1}{4}+\frac{1}{-j2}+\frac{1}{2}\right) \qquad j = \frac{1}{-j}$$

$$G_{12} = G_{21} = \frac{1}{4} \qquad I_1 = \frac{50\angle 0 \to V}{5 \to R} \qquad I_2 = \frac{50\angle 90 \to V}{2 \to R}$$

(Multiply off diagonal elements by −)



Prd: The node voltage equations are

$$\left(\frac{1}{6}+\frac{1}{j5}+\frac{1}{-j10}\right)V_1-\frac{1}{j5}V_2=\frac{10\angle 30}{6}$$

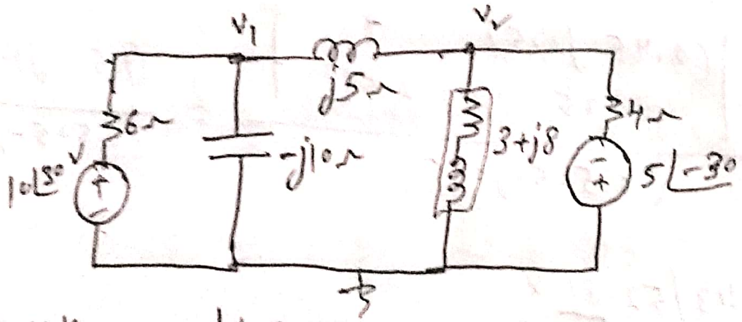
$$-\frac{1}{j5}V_1+\left(\frac{1}{j5}+\frac{1}{3+j8}+\frac{1}{4}\right)V_2=\frac{-5\angle -30}{4}$$

Draw the circuit.

Sol:

$$G_{11}=\left(\frac{1}{6}+\frac{1}{j5}+\frac{1}{-j10}\right) \qquad G_{12}=G_{21}=\frac{1}{j5}$$

$$G_{22}=\left(\frac{1}{j5}+\frac{1}{3+j8}+\frac{1}{4}\right) \qquad I_1=\frac{10\angle 30}{6} \quad I_2=\frac{-5\angle -30}{4}$$



Prd: The node voltage equations are,

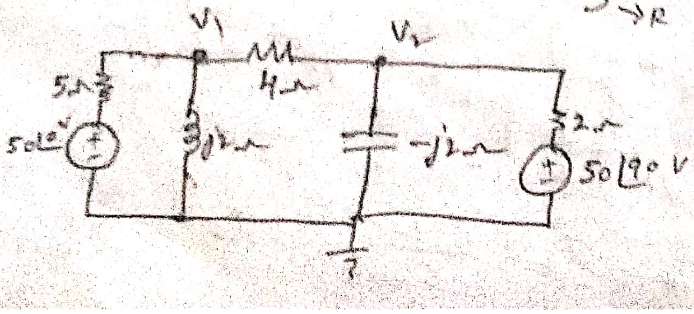
$$\left(\frac{1}{5}+\frac{1}{j2}+\frac{1}{4}\right)V_1-\frac{1}{4}V_2=\frac{50\angle 0}{5}$$

$$-\frac{1}{4}V_1+\left(\frac{1}{4}+\frac{1}{-j2}+\frac{1}{2}\right)V_2=\frac{50\angle 90}{2}$$

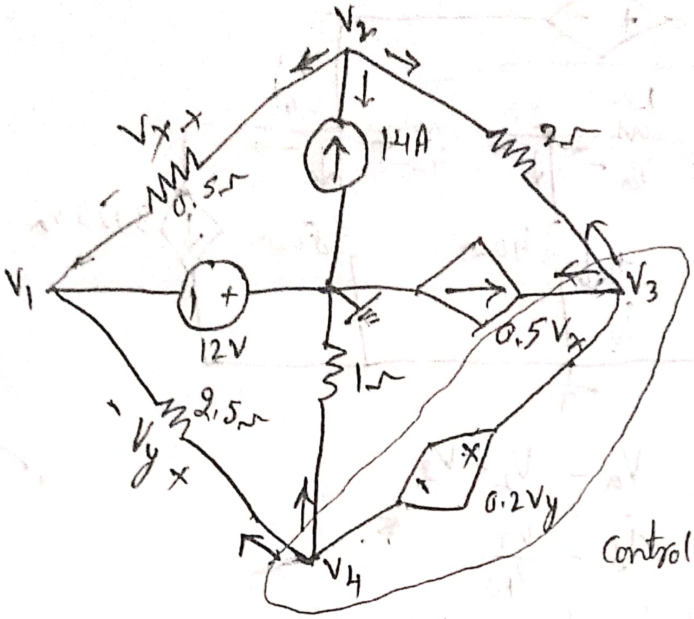
Sol:

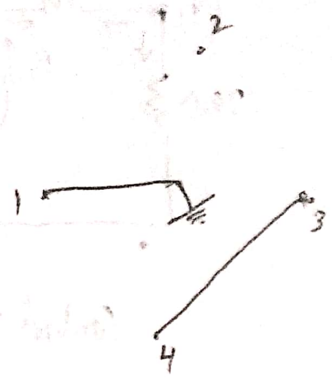
$$G_{11}=\left(\frac{1}{5}+\frac{1}{j2}+\frac{1}{4}\right) \qquad G_{12}=G_{21}=\frac{1}{4}$$

$$G_{22}=\left(\frac{1}{4}+\frac{1}{-j2}+\frac{1}{2}\right) \qquad I_1=\frac{50\angle 0 \to V}{5 \to R} \quad I_2=\frac{50\angle 90 \to V}{2 \to \Omega}$$



Q. Find the voltages V_1, V_2, V_3 & V_4 for the circuit shown in figure.





Control variable, $V_x = V_2 - V_1$
$V_y = V_4 - V_1$

Sol:

KVL to non-essential node 1: $0 - V_1 = 12 \Rightarrow V_1 = -12$

KVL to supernode 3 & 4: $V_3 - V_4 = 0.2(\underbrace{V_4 + 12}_{V_y})$

KCL to essential node 2: $\left(\frac{V_2 + 12}{0.5}\right) - 14 + \left(\frac{V_2 - V_3}{2}\right) = 0$

KCL to supernode 3 & 4: $\left(\frac{V_3 - V_2}{2}\right) - 0.5(\underbrace{V_2 + 12}_{V_x}) + \frac{(V_4 + 12)}{2.5} + \frac{V_4}{1} = 0$

$$(0)V_2 + (1)V_3 + (-1 - 0.2)V_4 = 0.2 \times 12$$

$$(0.5^{-1} + 2^{-1})V_2 + (-2^{-1})V_3 + (0)V_4 = -12 \times 0.5^{-1} + 14$$

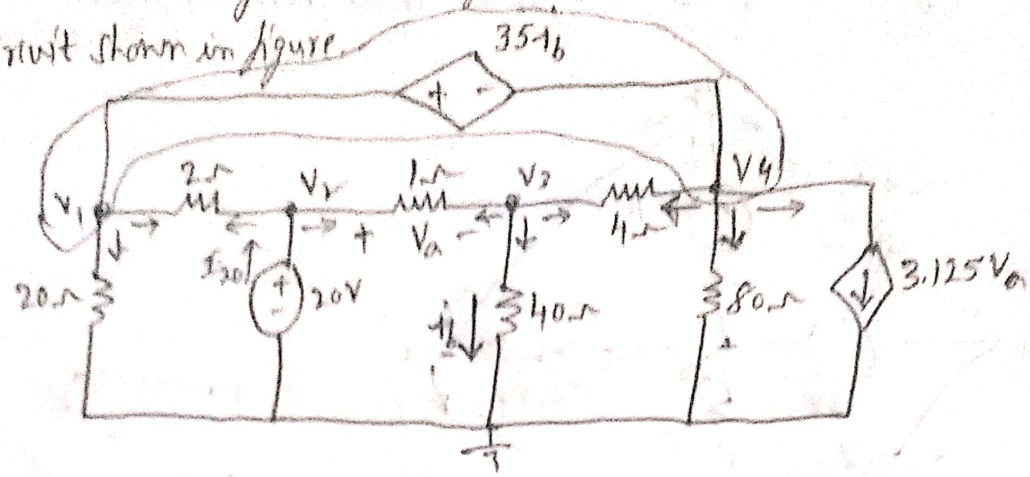
$$(-2^{-1} - 0.5)V_2 + (2^{-1})V_3 + (2.5^{-1} + 1^{-1})V_4 = 0.5 \times 12 - 12 \times 2.5^{-1}$$

$V_2 = -4\,V$ $\quad$ $V_1 = -12\,V$

$V_3 = 0\,V$

$V_4 = -2\,V$

Q. Use the node-voltage method to find the power developed by the 20V source in the circuit shown in figure.



Sol:

Control variable, $V_a = V_2 - V_3$

$$i_b = \frac{V_3}{40}$$

KVL to non-essential node 2: $V_2 = 20$

KVL to supernode 1&4: $V_1 - V_4 = 35\left(\frac{V_3}{40}\right)$

KCL to supernode 1&4: $\left(\frac{V_1}{20}\right) + \left(\frac{V_1 - 20}{2}\right) + \left(\frac{V_4 - V_3}{4}\right) + \left(\frac{V_4}{80}\right) + 3.125\underbrace{(20 - V_3)}_{V_a} = 0$

KCL to essential node 3: $\left(\frac{V_3}{40}\right) + \left(\frac{V_3 - 20}{1}\right) + \left(\frac{V_3 - V_4}{4}\right) = 0$

$$(1)V_1 + (-35\times 40^{-1})V_3 + (-1)V_4 = 0$$

$$(20^{-1} + 2^{-1})V_1 + (-4^{-1} - 3.125)V_3 + (4^{-1} + 80^{-1}) = +10 - 3.125\times 20$$

$$(0)V_1 + (40^{-1} + 1^{-1} + 4^{-1})V_3 + (-4^{-1}) = 20$$

$V_1 = -20.25V$

$V_2 = 20V$

$V_3 = 10V$

$V_4 = -29V$

$I_{20V} = I_{2\Omega} + I_{1\Omega}$

$= \left(\frac{V_2 - V_1}{2}\right) + \left(\frac{V_2 - V_3}{1}\right)$

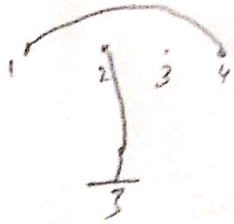
$= 20.125 + 10$

$= 30.125$

$P_{20V} = 20 \times I_{20}$

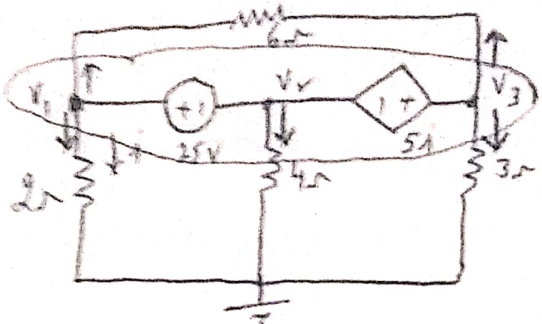
$= 20 \times 30.125$

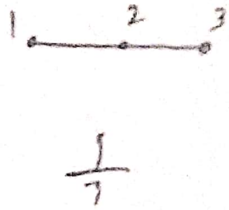
$= 602.5W$



b. Find the node voltages V_1, V_2 and V_3 in circuit diagram shown using nodal analysis (6 Marks)

Sol:





$i = V_1/2$

KVL to Supernode 1,2,3 : $V_1 - V_2 = 25$

(Two KVL for supernode since two voltage sources are merging)

$V_3 - V_2 = 5\left(\frac{V_1}{2}\right) \Rightarrow 2V_3 - 2V_2 = 5V_1$

KCL to supernode 1,2,3 :

(Applying KCL to node 1, 2, 3 simultaneously)

$$\left(\frac{V_1}{2}\right) + \left(\frac{V_1 - V_3}{6}\right) + \left(\frac{V_2}{4}\right) + \left(\frac{V_3 - V_1}{6}\right) + \frac{V_3}{3} = 0$$

Sorting the above equations

$$(1)V_1 + (-1)V_2 + (0)V_3 = 25$$

$$(-5)V_1 + (-2)V_2 + (2)V_3 = 0$$

$$(2^{-1} + 6^{-1} - 6^{-1})V_1 + (4^{-1})V_2 + (-6^{-1} + 6^{-1} + 3^{-1})V_3 = 0$$

$V_1 = 7.608\,V$

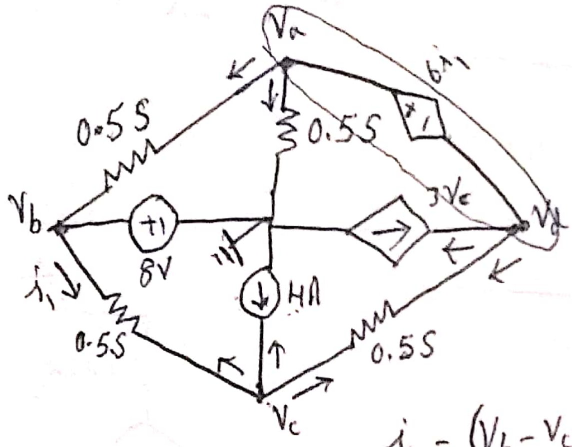
$V_2 = -17.39\,V$

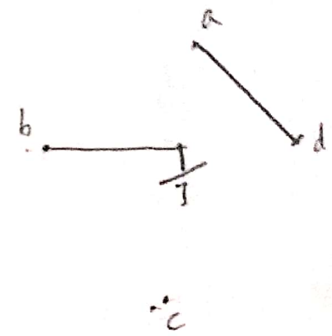
$V_3 = 1.63\,V$

c. Problem on resonance. It will come under module 3 of 1BEE32. It is solved there.

c) Find the node Voltages for the Circuit shown in figure using nodal analysis.
(6 marks)

S = Siemens or ℧





Sol:

$i_1 = (V_b - V_c) \times 0.5$

KVL to nonessential node b : $V_b = 8$ (In the next equations substitute 8 in place of V_b)

KVL to super node a & d : $V_a - V_d = 6\{\underbrace{(\overset{V_b}{8} - V_c) \times 0.5}_{i_1}\}$ — ①

KCL to super node a & d : $(V_a - 8) \times 0.5 + V_a \times 0.5 - 3V_c + (V_d - V_c)0.5 = 0$ — ②

KCL to essential node c : $(V_c - 8) \times 0.5 - 4 + (V_c - V_d) \times 0.5 = 0$ — ③

Sorting equations ①②③ :

$(1)V_a + (-6 \times 0.5)V_c + (-1)V_d = 6 \times 8 \times 0.5$

$(0.5+0.5)V_a + (-3-0.5)V_c + (0.5)V_d = 8 \times 0.5$

$(0)V_a + (0.5+0.5)V_c + (-0.5)V_d = 8 \times 0.5 + 4$

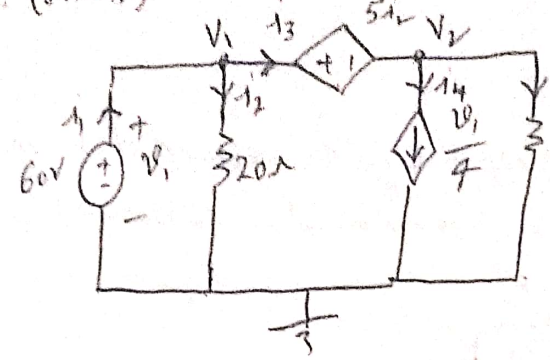
$V_a = 16\ V$ $\quad V_b = 8\ V$

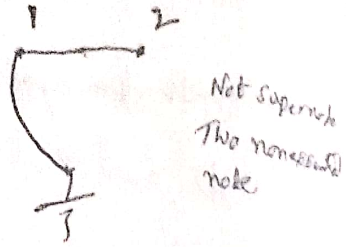
$V_c = 1.6\ V$

$V_d = -12.8\ V$

c. In the network shown in figure, solve for all the branch currents using nodal analysis and also show that the sum of power absorbed/delivered by all branches is zero. (6 Marks)

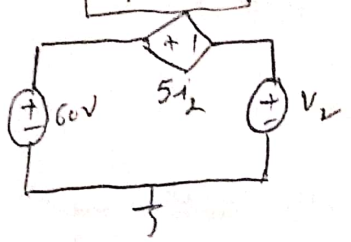
Sol:





Not Supernode
Two nonessential node

$$\boxed{V_1 = 60}$$



$$-60 + 5i_2 + V_2 = 0$$

$$i_2 = \frac{V_1}{20} = \frac{60}{20} = 3$$

$$-60 + 5 \times 3 + V_2 = 0$$

$$\boxed{V_2 = 45}$$

Branch currents, $i_2 = 3A$

$$i_4 = \frac{V_1}{4} = 15A$$

$$i_5 = \frac{V_2}{5} = 9A$$

By KCL, $i_3 = i_4 + i_5 = 24A$

$$i_1 = i_2 + i_3 = 27A$$

Power absorbed / delivered

$P = \frac{V^2}{R} = V^2 G$ $P_{absorbed\,(20\Omega)} = V_1^2 \times 20^{-1} = 60^2 \times 20^{-1} = 180W$

$P_{absorbed\,(5\Omega)} = V_2^2 \times 5^{-1} = 45^2 \times 5^{-1} = 405W$

$P = \pm VI$ $P_{absorbed\,(60V)} = -V_1 i_1 = -60 \times 27 = -1620W$, Power delivered $= +1620W$

↳ Current is entering '−' terminal

$P_{absorbed}$ (dependent source $5i_2$) $= +(5i_2)(i_3) = 5 \times 3 \times 24 = 360W$

$P_{absorbed}$ (dependent source $\frac{V_1}{4}$) $= V_2 \times \frac{V_1}{4} = 45 \times \frac{60}{4} = 675W$

sum of power absorbed $= 180 + 405 - 1620 + 360 + 675 = 0$

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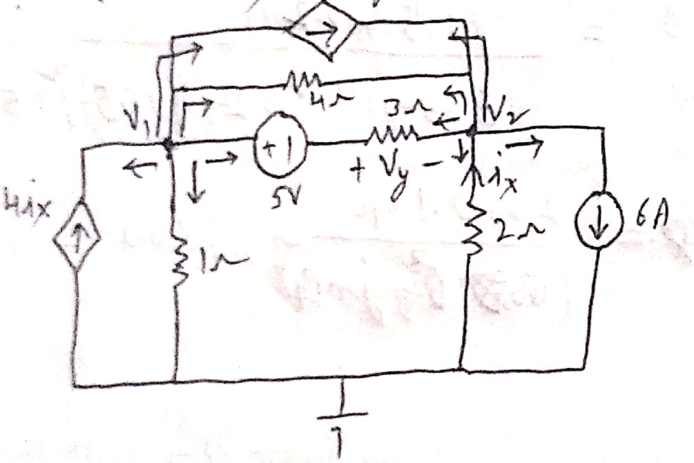
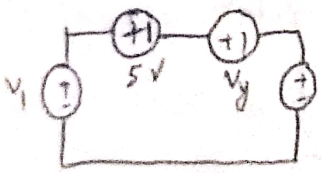
Module - 1 (15EE32, June/July 2019)

1 a. Write a system of nodal equations for the circuit shown in figure using nodal voltages V_1 & V_2 as the variables. What power is furnished by the $5V_y$ dependent source? (10 Marks)

Sol:



$$i_x = \frac{0 - V_2}{2} = \frac{-V_2}{2}$$



$$-V_1 + 5 + V_y + V_2 = 0$$

$$V_y = V_1 - V_2 - 5$$

KCL to essential node 1:

$$\underbrace{-4\left(-\frac{V_2}{2}\right)}_{i_x} + \frac{V_1}{1} + \left(\frac{V_1 - V_2}{4}\right) + 5(\underbrace{V_1 - V_2 - 5}_{V_y}) + \left(\frac{V_1 - V_2 - 5}{3}\right) = 0$$

KCL to essential node 2:

$$\left(\frac{V_2}{2}\right) + 6 + \left(\frac{V_2 - V_1 + 5}{3}\right) + \left(\frac{V_2 - V_1}{4}\right) - 5(\underbrace{V_1 - V_2 - 5}_{V_y}) = 0$$

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$$\left(1^{-1}+4^{-1}+5+3^{-1}\right) V_1 + \left(2-4^{-1}-5-3^{-1}\right) V_2 = +25+5\times 3^{-1}$$

$$\left(-3^{-1}-4^{-1}-5\right) V_1 + \left(2^{-1}+3^{-1}+4^{-1}+5\right) V_2 = -6-5\times 3^{-1}-25$$

$$V_1 = 2.25\,V$$

$$V_2 = -3.3\,V$$

$P_{absorbed}$ () $= V\,I$

$5v_y$

$$= (V_1 - V_2)(5V_y)$$

$$= (V_1 - V_2)\{5(V_1 - V_2 - 5)\}$$

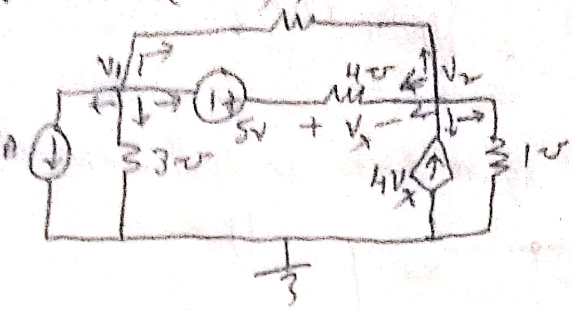
$$= (2.25+3.3)\times\{5(2.25+3.3-5)\}$$

$$= (5.55)(2.75)$$

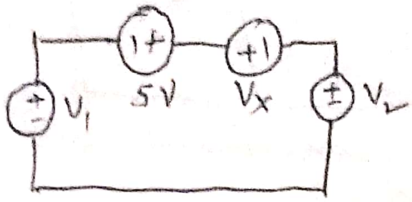
$$= 15.26\,W$$

1.a. Set up nodal equations for the circuit shown in figure and then find the power supplied by 5V source. (5 marks)

Sol:



To get V_x in terms of node voltages



$$-V_1 - 5 + V_x + V_2 = 0$$

$$V_x = V_1 - V_2 + 5.$$

KCL to essential node 1: $+5 + V_1 \times 3 + (V_1 - V_2) \times 2 + (V_1 - V_2 + 5) \times 4 =$

KCL to essential node 2: $-4\underbrace{(V_1 - V_2 + 5)}_{V_x} + V_2 \times 1 + (V_2 - V_1 - 5) \times 4 + (V_2 - V_1) \times$

$$(3+2+4)V_1 + (-2-4)V_2 = -5-20$$

$$(-4-4-2)V_1 + (4+1+4+2)V_2 = +20+20$$

$$V_1 = -0.897\ V$$

$$V_2 = 2.820\ V$$

$$(P)_{\substack{absorbed \\ (5V)}} = -(V)(I)$$

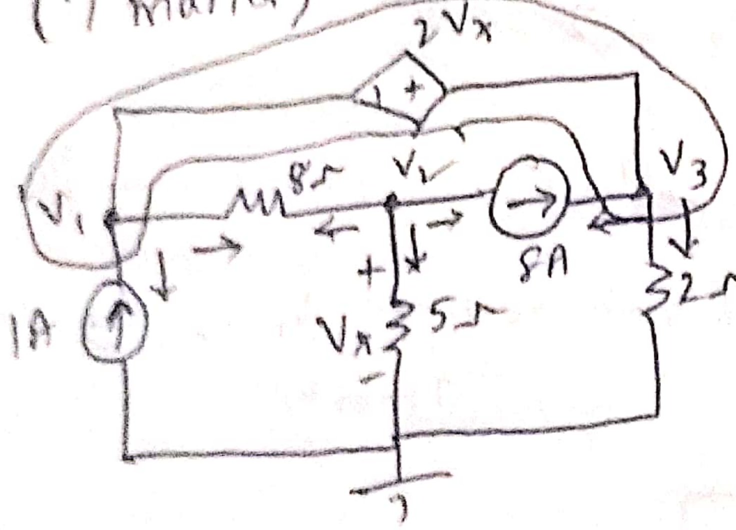
$\hookrightarrow$ current is entering '−' terminal of 5V

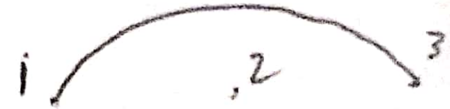
$$= -\underbrace{(5)}_{V}\underbrace{(V_1 - V_2 + 5) \times 4}_{I}$$

$$= -5 \times (-0.897 - 2.82 + 5) \times 4 = -25.66W$$

$P_{delivered\,(5V)} = +25.66W$

6. Determine the voltage V_x in the circuit shown in figure using nodal analysis method (7 marks)





$$V_x = V_2$$

KVL to supernode 1 & 3 : $V_3 - V_1 = 2V_2$ $\rightarrow V_x$

KCL to supernode 1 & 3 : $-1 + \frac{(V_1 - V_2)}{8} - 8 + \frac{V_3}{2} = 0$

KCL to essential node 2 : $\left(\frac{V_2 - V_1}{8}\right) + 8 + \left(\frac{V_2}{5}\right) = 0$

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$$(-1)V_1 + (-2)V_2 + (1)V_3 = 0$$

$$(8^{-1})V_1 + (-8^{-1})V_2 + (2^{-1})V_3 = 9$$

$$(-8^{-1})V_1 + (8^{-1}+5^{-1})V_2 + (0)V_3 = -8$$

$$V_1 = 31.76\,V$$

$$V_2 = -12.4\,V$$

$$V_3 = 6.96\,V$$

$$V_x = V_2 = -12.4\,V$$